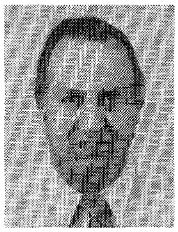

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Aspects of the Calibration of a Single Six-Port Using a Load and Offset Reflection Standards

G. P. RIBLET, MEMBER, IEEE, AND E. R. BERTIL HANSSON

Abstract —**In this contribution some aspects of the calibration of a single six-port using a load and offset reflection standards are discussed. The applicability of the methods developed is demonstrated by the successful calibration of several six-ports including one consisting of a directional coupler plus a symmetrical five-port junction.**

I. Introduction

Although the theory for the calibration of six-ports using the dual six-port method is well developed at this time [1], the calibration of these devices using offset reflection standards is attractive, particularly in a typical laboratory environment. Problems with the latter are a) the absence of simple closed form expressions for the calibration constants [2] and b) insight into what standards to choose to optimize the calibration over a given frequency interval. This contribution attempts to remedy this situation. A third problem of much practical significance relates to the transferability of the calibration. It is shown how the calibration constants can be normalized in such a way that a six-port can be recalibrated with a good load on the output without the need to go through a full calibration procedure whenever the device is used in a different experimental configuration.

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II. Choice of Offset Standards

If P_R is the power measured by the reference detector and P_i, $i=1,2,3$ the powers measured by the other three detectors attached to the six-port (see Fig. 1), then the power ratios P_i/P_R can be written

$$P_i/P_R = Y^i \frac{1+2X_i|\Gamma_u|\cos(\phi_{x_i}+\phi_u)+X_i^2|\Gamma_u|^2}{1+2Z|\Gamma_u|\cos(\phi_z+\phi_u)+Z^2|\Gamma_u|^2}, \quad i=1,2,3 \quad (1)$$

where $|\Gamma_u|$ is the magnitude and ϕ_u the phase of the reflection coefficient to be measured [3]. The other quantities are calibration constants of the six-port. The term Y^i, $i=1,2,3$ can be determined from a measurement of a very good load or a sliding load on the output. If the reference coupler has infinite directivity and the six-port is perfectly matched, then Z will be zero and only terms in the numerator will appear. In general, these conditions will be approximately fulfilled so that Z will be small and the denominator will be close to one. Let us assume that Z, ϕ_z are known (a procedure is given in the next section for

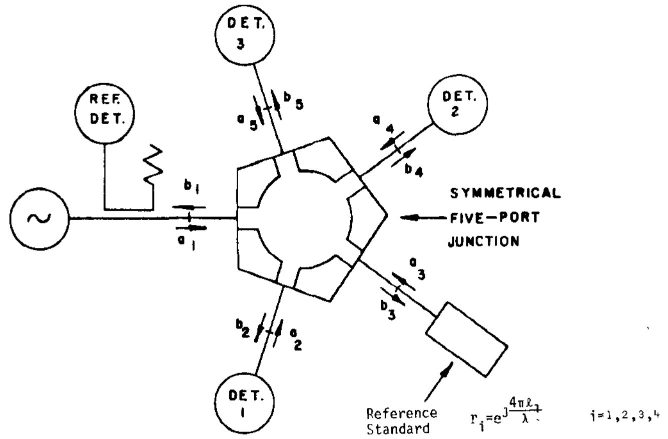

Fig. 1. Experimental configuration for the calibration of a six-port consisting of a directional coupler and a symmetrical five-port junction by using a load and four offset unit amplitude reflection standards.

determining them), then we can define known quantities $(P_i/P_R)'$ such that

$$(P_i/P_r)' = 1 + 2X_i|\Gamma_u|\cos(\phi_{x_i} + \phi_u) + X_i^2|\Gamma_u|^2, \quad i = 1,2,3. \quad (2)$$

Notice the symmetry between the magnitude $|\Gamma_u|$ and phase ϕ_u of the reflection coefficient and the calibration terms X_i and ϕ_{x_i} in the above equation. Six-port theory tells us what $X_i, \phi_{x_i}\, i = 1,2,3$ to choose in order to determine $|\Gamma_u|, \phi_u$ accurately everywhere. Using the symmetry of the above equation we may use the same theory to determine what $|\Gamma_u(j)|, \phi_u(j)$ to choose in order to determine $X_i, \phi_{x_i}\, i = 1,2,3$ accurately. In particular with $|\Gamma_u| = 1$ we would need three standards with reflection phases separated by 120° at midband. The calibration constants $X_i\cos\phi_{x_i}$, $X_i\sin\phi_{x_i}$ will be given linearly in terms of the known quantities $|\Gamma_u(j)|\cos(\phi_u(j))$, $|\Gamma_u(j)|\sin(\phi_u(j))\, j = 1,2,3$. Clearly the calibration constants will now be determined unambiguously and accurately provided Z, ϕ_z are known.

Unfortunately, using three standards does not give us the possibility of determining what Z and ϕ_z actually are. For this we need a fourth reflection standard. The above arguments make plausible the choice of four unit magnitude standards with phases separated by 90° intervals. This choice has been found to work well in practice. Two possibilities have been found to be of practical interest. For a six-port which terminates in a TEM transmission line such as 7-mm line, it has been found to be appropriate to use an open circuit, a short circuit, and two offset open circuits—one l unit shorter and one l unit longer than the first open circuit in order to obtain an accurate calibration over as great a bandwidth as possible. For a six-port which terminates in a transmission line with a cutoff frequency f_c, such as waveguide, it is appropriate to use four short circuits spaced at l units. The numerator calibration constants $X_i\cos\phi_{x_i}$, $X_i\sin\phi_{x_i}$, and $i = 1,2,3$ will be related to the normalized powers $(P_i/P_R)'_j$, $j = 1,2,3,4$ associated with one detector but the four different standards by a 2×4 matrix. In the case of a TEM line, the matrix is

$$\begin{bmatrix} X_i\cos\phi_{x_i} \\ X_i\sin\phi_{x_i} \end{bmatrix} = 1/4 \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -\dfrac{1}{\sin(4\pi l/\lambda)} & \dfrac{1}{\sin(4\pi l/\lambda)} \end{bmatrix} \cdot \begin{bmatrix} (P_i/P_R)'_1 \\ (P_i/P_R)'_2 \\ (P_i/P_R)'_3 \\ (P_i/P_R)'_4 \end{bmatrix} i = 1,2,3. \quad (3)$$

Clearly the calibration procedure will not work if $\sin(4\pi l/\lambda) = 0$ in the denominator or if $4\pi l/\lambda = n\pi$. In particular, a condition on l which insures this won't happen in the band from f_1 to f_2 is that $\pi/2 - 4\pi l/\lambda_1 = 4\pi l/\lambda_2 - \pi/2$ or

$$l = 75/(f_1 + f_2) \quad (4)$$

where f_1 and f_2 are in gigahertz, l is in millimeters, and λ_1, λ_2 are the corresponding wavelengths. For a calibration over a 5/1 bandwidth, the terms $-1/\sin(4\pi l/\lambda)$, $1/\sin(4\pi l/\lambda)$ become at the band edges 1/2 their midband values so that this calibration procedure should work over bandwidths greater than 5/1. In the case of a transmission line with finite cutoff frequency f_c

$$\begin{bmatrix} X_i\cos\phi_{x_i} \\ X_i\sin\phi_{x_i} \end{bmatrix} = \frac{1}{4\sin^2\left(\dfrac{4\pi l}{\lambda_g}\right)} \cdot \begin{bmatrix} -\cos\dfrac{8\pi l}{\lambda g}, \cos\dfrac{4\pi l}{\lambda_g}, \cos\dfrac{8\pi l}{\lambda_g}, -\cos\dfrac{4\pi l}{\lambda_g} \\ -\sin\dfrac{8\pi l}{\lambda_g}, \sin\dfrac{4\pi l}{\lambda_g}, \sin\dfrac{8\pi l}{\lambda_g}, -\sin\dfrac{4\pi l}{\lambda_g} \end{bmatrix} \cdot \begin{bmatrix} (P_i/P_R)'_1 \\ (P_i/P_R)'_2 \\ (P_i/P_R)'_3 \\ (P_i/P_R)'_4 \end{bmatrix}. \quad (5)$$

If we wish to calibrate the six-port over the band f_1 to f_2, then the condition on l becomes $\pi/2 - 4\pi l/\lambda_{g1} = 4\pi l/\lambda_{g2} - \pi/2$ or

$$l = 150\cdot\left\{3f_2\cdot\sqrt{1-(f_c/f_2)^2} + f_1\cdot\sqrt{1-(f_c/f_1)^2}\right\}^{-1} \quad (6)$$

where again f_1 and f_2 are in gigahertz while l is in millimeters. In this case the $\sin^2(4\pi l/\lambda_g)$ term in the denominator will be nonzero in the band f_1 to f_2.

III. Closed Form Expressions for Z, ϕ_z

Let $R_0(i), R_1(i), R_2(i), R_3(i), i = 1,2,3$ be the measured normalized powers at the three detectors for four unit magnitude offset standards with reflection phase angles

$\phi_0, \phi_1, \phi_2, \phi_3$, respectively, and obtained by dividing (P_i/P_R) by Y_i in 1). It follows from (1) that

$$\begin{bmatrix} R_1(i) & R_1(i)\cos\phi_1 & -R_1(i)\sin\phi_1 \\ R_2(i) & R_2(i)\cos\phi_2 & -R_2(i)\sin\phi_2 \\ R_3(i) & R_3(i)\cos\phi_3 & -R_3(i)\sin\phi_3 \end{bmatrix} \begin{bmatrix} 1+Z^2 \\ 2Z\cos\phi_z \\ 2Z\sin\phi_z \end{bmatrix} = \begin{bmatrix} 1 & \cos\phi_1 & -\sin\phi_1 \\ 1 & \cos\phi_2 & -\sin\phi_2 \\ 1 & \cos\phi_3 & -\sin\phi_3 \end{bmatrix} \begin{bmatrix} 1+X_i^2 \\ 2X_i\cos\phi_{x_i} \\ 2X_i\sin\phi_{x_i} \end{bmatrix} \quad (7)$$

$$\therefore \begin{bmatrix} 1+X_i^2 \\ 2X\cos\phi_{xi} \\ 2X\sin\phi_{x_i} \end{bmatrix} = [B]\cdot \begin{bmatrix} 1+Z^2 \\ 2Z\cos\phi_z \\ 2Z\sin\phi_z \end{bmatrix} \quad (8)$$

where

$$[B] = \{\sin(\phi_1-\phi_2)+\sin(\phi_2-\phi_3)+\sin(\phi_3-\phi_1)\}^{-1} \cdot \begin{bmatrix} \sin(\phi_2-\phi_3) & \sin(\phi_3-\phi_1) & \sin(\phi_1-\phi_2) \\ \sin\phi_3-\sin\phi_2 & \sin\phi_1-\sin\phi_3 & \sin\phi_2-\sin\phi_1 \\ \cos\phi_3-\cos\phi_2 & \cos\phi_1-\cos\phi_3 & \cos\phi_2-\cos\phi_1 \end{bmatrix} \cdot \begin{bmatrix} R_1(i) & \cos\phi_1 R_1(i) & -\sin\phi_1 R_1(i) \\ R_2(i) & \cos\phi_2 R_2(i) & -\sin\phi_2 R_2(i) \\ R_3(i) & \cos\phi_3 R_3(i) & -\sin\phi_3 R_3(i) \end{bmatrix}.$$

The values so obtained for $1+X_i^2$, $X_i\cos\phi_{x_i}$, $X_i\sin\phi_{x_i}$ can be substituted into the remaining equation

$$\begin{aligned} &R_0(i)(1+Z^2)+R_0(i)\cos\phi_0 2Z\cos\phi_z \\ &\quad - R_0(i)\sin\phi_0 2Z\sin\phi_z \\ &\quad = 1+X_i^2+\cos\phi_0\cdot 2X_i\cos\phi_{x_i} \\ &\quad - \sin\phi_0 2X_i\sin\phi_{x_i} \end{aligned} \quad (9)$$

to obtain three equations in the quantities $1+Z^2$, $Z\cos\phi_z$, $Z\sin\phi_z$. Unfortunately, it is not possible to obtain a linear solution for $Z\cos\phi_z$ and $Z\sin\phi_z$ as was the case in Section II. However, a quadratic equation for Z can be obtained and the root with $Z<1$ selected. In matrix notation these equations can be written

$$[A]\cdot \begin{bmatrix} 1+Z^2 \\ 2Z\cos\phi_z \\ 2Z\sin\phi_z \end{bmatrix} = 0 \quad (10)$$

where the components of the 3×3 matrix A are given by

$$\begin{bmatrix} R_1(i)-R_0(i) & R_2(i)-R_0(i) & R_3(i)-R_0(i) \\ R_1(i)\cos\phi_1-R_0(i) & R_2(i)\cos\phi_2-R_0(i) & R_3(i)\cos\phi_3-R_0(i) \\ -R_1(i)\sin\phi_1 & -R_2(i)\sin\phi_2 & -R_3(i)\sin\phi_3 \end{bmatrix} \cdot \begin{pmatrix} \sin\phi_2-\sin\phi_3-\sin(\phi_2-\phi_3) \\ \sin\phi_3-\sin\phi_1-\sin(\phi_3-\phi_1) \\ \sin\phi_1-\sin\phi_2-\sin(\phi_1-\phi_2) \end{pmatrix} = \begin{pmatrix} A_{i1} \\ A_{i2} \\ A_{i3} \end{pmatrix}, \quad i=1,2,3. \quad (11)$$

From (10)

$$\begin{aligned} A_{11}(Z+1/Z)/2+A_{12}\cos\phi_z+A_{13}\sin\phi_z&=0 \\ A_{21}(Z+1/Z)/2+A_{22}\cos\phi_z+A_{23}\sin\phi_z&=0 \\ A_{31}(Z+1/Z)/2+A_{32}\cos\phi_z+A_{33}\sin\phi_z&=0. \end{aligned} \quad (12)$$

A solution for ϕ_z can be generated in three ways, i.e., by eliminating $(Z+1/Z)/2$ from the first two equations, the last two equations, or the first and last equations. We find that

$$\begin{aligned} \phi_z &= \tan^{-1}\left\{\frac{A_{21}\cdot A_{12}-A_{11}\cdot A_{22}}{A_{11}\cdot A_{23}-A_{21}\cdot A_{13}}\right\} \\ &= \tan^{-1}\left\{\frac{A_{21}\cdot A_{32}-A_{31}\cdot A_{22}}{A_{31}\cdot A_{23}-A_{21}\cdot A_{33}}\right\} \\ &= \tan^{-1}\left\{\frac{A_{31}\cdot A_{12}-A_{11}\cdot A_{32}}{A_{11}\cdot A_{33}-A_{31}\cdot A_{13}}\right\}. \end{aligned} \quad (13)$$

The 180°-phase ambiguity that results from (13) is resolved by the requirement that $(Z+1/Z)/2$ be positive in (12). Once ϕ_z has been found, any one of the three equations in (12) will yield a quadratic equation in Z. The three equations are of the form

$$(Z+1/Z)/2 = X \quad (14)$$

where

$$\begin{aligned} X &= -\frac{A_{12}\cos\phi_z+A_{13}\sin\phi_z}{A_{11}} \\ &= -\frac{A_{22}\cos\phi_z+A_{23}\sin\phi_z}{A_{21}} \\ &= -\frac{A_{32}\cos\phi_z+A_{33}\sin\phi_z}{A_{31}}. \end{aligned} \quad (15)$$

The trigonometric identity $1/\sin\theta = \{\tan(\theta/2)+1/\tan\cdot(\theta/2)\}/2$ may be used to solve for Z. In particular $\theta = \sin^{-1}(1/X)$, $Z=\tan(\theta/2)$. There will be one solution for Z less than 1 and one greater than 1 depending on whether θ is chosen in the first or second quadrant. It is necessary to choose θ in the first quadrant and not the second to make $Z<1$ since Z is close to zero for most six-ports. A particular choice of the three solutions given in (13) and in (14) has been found in practice to insure accurate results. In (13), ϕ_z can be considered to be the phase angle of any one of three two-dimensional vectors with x and y components given by the denominators and numerators, respectively,

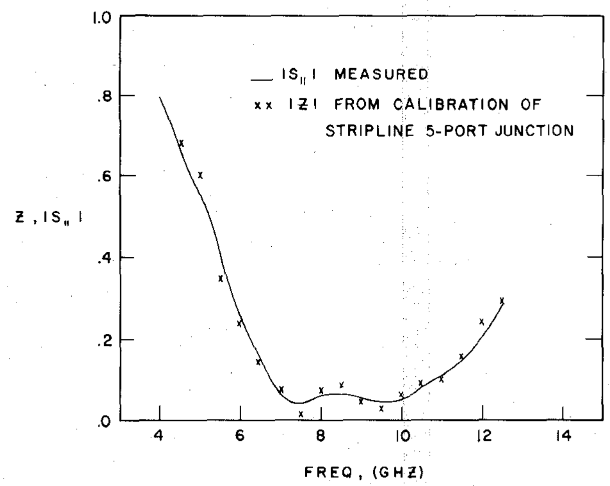

Fig. 2. Comparison between the experimental values of Z obtained with this calibration procedure and the measured values of $|S_{11}|$ for a stripline five-port junction. The values should be the same if a coupler with infinite directivity is used.

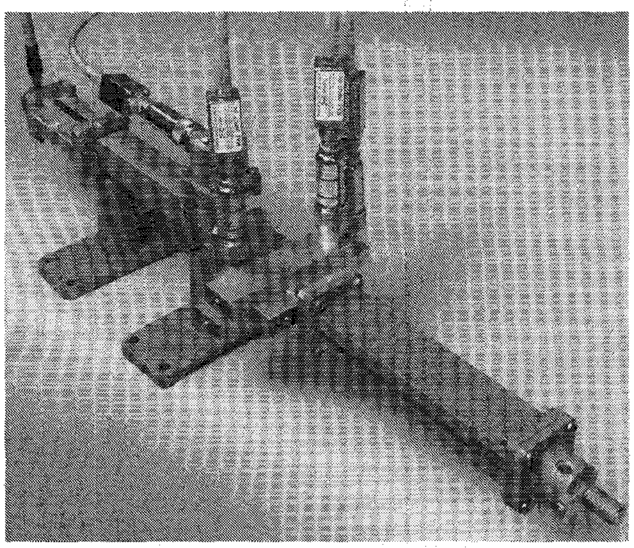

Fig. 3. Picture of waveguide WR90 six-port based on circular coupling holes.

and with magnitudes

$$S_1 = \left((A_{21}A_{12} - A_{11}A_{22})^2 + (A_{11}\cdot A_{23} - A_{21}\cdot A_{13})^2\right)^{1/2}$$
$$S_2 = \left((A_{21}\cdot A_{32} - A_{31}\cdot A_{22})^2 + (A_{31}\cdot A_{23} - A_{21}\cdot A_{33})^2\right)^{1/2}$$
$$S_3 = \left((A_{31}\cdot A_{12} - A_{11}\cdot A_{32})^2 + (A_{11}\cdot A_{33} - A_{31}\cdot A_{13})^2\right)^{1/2}. \tag{16}$$

We have taken ϕ_z to be the phase angle of the vector with the largest magnitude. Similarly, in (14), X is determined using the expression with the largest denominator A_{11}, A_{21}, or A_{31}. A simple proof that accurate results for Z can always be obtained has yet to be found. However, this method has been used successfully to calibrate several six-ports including ones consisting of a directional coupler

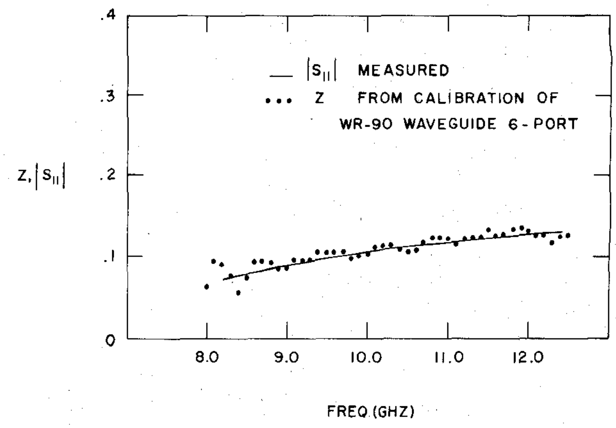

Fig. 4. Comparison between values of Z obtained from the calibration procedure for the device of Fig. 3 using four offset short circuits and the measured values of $|S_{11}|$ at the output port.

followed by a nearly matched symmetrical five-port junction [4]. If the directivity of the coupler is infinite, then it can be shown that $Z = |S_{11}|$ —the magnitude of the input reflection coefficient of the reciprocal five-port junction. As a check on the calibration procedure, $|S_{11}|$ of the five-port was measured and compared with the values of Z obtained with the above calibration procedure when a coupler with better than 30-dB directivity was used. The agreement given in Fig. 2 is quite good. In order to demonstrate that the accuracy of the calibration is in no way related to the use of a symmetrical five-port junction, we have also successfully calibrated a waveguide six-port based on circular coupling holes a picture of which is given in Fig. 3. In Fig. 4 a comparison is given between the values of Z obtained from the calibration when a coupler with better than 35-dB directivity was used and the measured values of $|S_{11}|$. Once Z and ϕ_z have been found, the other calibration constants must be accurately determined as explained in Section II.

IV. Recalibration Using a Very Good Load

The final section discusses a point of considerable practical significance—namely the possibility of recalibrating a six-port when it is used in a different experimental configuration without going through a full calibration procedure. In particular, it will be shown how this may be done with a very good load or with a sliding load under the assumption the detector ports are isolated from one another. This condition is very well fulfilled in the case of the six-port of Fig. 3 and less well fulfilled in the case of the six-port of Fig. 1. If it were necessary to fully recalibrate a six-port whenever it is used in a new experimental configuration, it would, in the authors' opinion, severely limit the applicability of this device because of the operator skill required to perform the calibration. By storing the calibration constants in a form which allows for recalibration with a load, the main calibration could be performed at a facility set up for this purpose and then stored permanently in ROM memory, for example.

The basic problem relates to the calibration constants Y^i in (1). If different detectors are used or if different amplifier settings are used than those used to perform the initial calibration, then these coupling constants will be different. This would in theory require a full recalibration. However, these constants are determined uniquely by the powers measured with a load on the output so that it should be possible to express the basic calibration in such form that the six-port can be readily recalibrated with a load. In order to see how this can be done, it is useful to use the $[C]$ and $[D]$ matrix notation introduced by Cronson and Susman [5]. These matrices are four-dimensional matrices which are the inverse of each other. The elements of the $[D]$ matrix are used to determine the components of the reflection coefficient while the elements of $[C]$ are directly related to the calibration constants given in (1). In particular

$$[C]=\begin{bmatrix} Y^{(1)} & 2Y^{(1)}\cdot X_1\cos\phi_{x_1} & 2Y^{(1)}\cdot X_1\sin\phi_{x_1} & Y^{(1)}X_1^2 \\ Y^{(2)} & 2Y^{(2)}\cdot X_2\cos\phi_{x_2} & 2Y^{(2)}\cdot X_2\sin\phi_{x_2} & Y^{(2)}X_2^2 \\ Y^{(3)} & 2Y^{(3)}\cdot X_3\cos\phi_{x_3} & 2Y^{(3)}\cdot X_3\sin\phi_{x_3} & Y^{(3)}X_3^2 \\ l & 2Z\cos\phi_z & 2Z\sin\phi_z & Z^2 \end{bmatrix}. \tag{17}$$

The problem becomes how to determine new components for $[D]$ without first having to determine the new components of $[C]$ and then inverting a four-dimensional matrix. Notice that $Y^{(1)}$ multiplies the first row of $[C]$, $Y^{(2)}$ the second row, and $Y^{(3)}$ the third row. It can be shown that if a number multiplies a row of a matrix then it will divide the corresponding column of the inverse matrix. Consequently, the matrix $[D']$ which must be stored is the inverse of

$$[C']=\begin{bmatrix} l & 2X_1\cos\phi_{x_1} & 2X_1\sin\phi_{x_1} & X_1^2 \\ l & 2X_2\cos\phi_{x_2} & 2X_2\sin\phi_{x_2} & X_2^2 \\ l & 2X_3\cos\phi_{x_3} & 2X_3\sin\phi_{x_3} & X_3^2 \\ l & 2Z\cos\phi_z & 2Z\sin\phi_z & Z^2 \end{bmatrix}. \tag{18}$$

The elements of $[D]$ can then be calculated from those of $[D']$ by simple division operations. It can be shown that the elements of $[D']$ satisfy the following two equations:

$$D'_{21}+D'_{22}+D'_{23}+D'_{24}=0$$
$$D'_{31}+D'_{32}+D'_{33}+D'_{34}=0. \tag{19}$$

As a consequence the number of matrix calibration constants which must be stored is reduced from 11 to 9. The components of the reflection coefficient can be written as

$$\Gamma\cos\theta=\frac{\frac{D'_{21}}{D'_{14}}(\bar{P}_1-1)+\frac{D'_{22}}{D'_{14}}(\bar{P}_2-1)+\frac{D'_{23}}{D'_{14}}(\bar{P}_3-1)}{\frac{D'_{11}}{D'_{14}}\bar{P}_1+\frac{D'_{12}}{D'_{14}}\bar{P}_2+\frac{D'_{13}}{D'_{14}}\bar{P}_3+1}$$

and

$$\Gamma\sin\theta=\frac{\frac{D'_{31}}{D'_{14}}(\bar{P}_1-1)+\frac{D'_{32}}{D'_{14}}(\bar{P}_2-1)+\frac{D'_{33}}{D'_{14}}(\bar{P}_3-1)}{\frac{D'_{11}}{D'_{14}}\bar{P}_1+\frac{D'_{12}}{D'_{14}}\bar{P}_2+\frac{D'_{13}}{D'_{14}}\bar{P}_3+1} \tag{20}$$

where $\bar{P}_i$ is the *ratio* of P_i/P_R to the value with a load on the output. An advantage of this formulation is that the reflection coefficient is forced to be small for measurements of *well*-matched devices, i.e., $\bar{P}_i \simeq 1$. This is a common application. The initial calibration procedure for a six-port used to measure Γ then becomes somewhat analogous to the initial calibration procedure for an SWR bridge used to measure return loss. In the first instance before measurements begin a calibration reading with a good load on the output must be made while for the second a calibration reading with an open or short circuit on the output must be made.

V. Summary

A simple calibration procedure for six-ports has been developed which allows them to be calibrated in a laboratory environment using a very good load and four unit magnitude offset reflection standards. The expressions for the calibration constants are explicit. The steps in the calibration procedure can be summarized as follows.

1) Choose the offset length l to conform to (4) or (6) depending on which is applicable.
2) Determine the calibration constants Y^i from the measurement of a very good load on the output by using (1).
3) Divide the power ratios (P_i/P_R) by Y^i to determine the normalized powers $R(i)$ used in (11).
4) Evaluate the denominator calibration terms ϕ_z, Z from (11), (13), (14), and (15).
5) Use these quantities to determine the normalized ratios $(P_i/P_R)'$ defined by (1) and (2).
6) Use (3) or (5) depending on which is applicable to determine the numerator calibration constants $X_i\cos\phi_{x_i}$, $X_i\sin\phi_{x_i}$.
7) Determine the 4×4 matrix $[C']$ from (18).
8) Invert $[C']$ to determine the 4×4 calibration matrix $[D']=[C']^{-1}$ whose entries are used to evaluate the reflection coefficient.

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Efficient Eigenmode Analysis for Planar Transmission Lines

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***Abstract*—A unified analysis for planar transmission lines is performed using the mode-matching technique. Exploiting the fact that the thickness of the metal coating (fins or strips) is usually very small in comparison to all other dimensions, the characteristic equations are formulated in a way which preserves the physical meaning of their individual terms. Thus, simplifications of far-reaching consequences can be introduced for all eigenmodes showing a cutoff frequency. It is shown in particular that the higher order modes can be derived approximately from the fundamental mode. Moreover, the dispersion relation of fin-lines can be given by a simple expression because the equivalent dielectric constant linearly depends on frequency. Both steps reduce the computer time by about two orders of magnitude in comparison to the spectral-domain method.**

I. Introduction

NUMEROUS PAPERS have appeared dealing with a rigorous solution of the dispersion problem of various planar transmission lines. Highly sophisticated techniques have been developed and applied, one of the most favorable being the spectral-domain method in conjunction with Ritz–Galerkin's method. Two references may stand for many investigations: [1], [2]. Common to all of these works is a time-consuming evaluation of the final relations. Hence, there are but few papers dealing with an application of the eigenmode analysis to circuit problems. This contribution deals with an approximate and efficient analysis of planar transmission lines and its application to fin-lines. Using the mode-matching technique, the final equations are formulated in a way which allows introducing some essential simplifications. The main difference to existing methods is a reduction in computer time of about two orders of magnitude. Hence, the analysis should be well suited for a computer-aided design of microwave planar circuits.

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II. Analysis

The structure which has been analyzed consists of an arbitrary number of metallic strips which are deposited on either side of a dielectric substrate. This planar circuit may be mounted either in the H-plane or in the E-plane of a rectangular box. Hence, the structure can be specialized to represent a microstrip line, coupled striplines, a slot line, a coplanar line, a microstrip line with tuning septums, a bilateral, unilateral, or antipodal fin-line, and a multislot fin-line. For explaining the calculation procedure, the cross section of the latter is shown in Fig. 1. The metallic strips are assumed to have finite thickness. This eliminates, on one hand, the existence of field singularities due to an edge condition while it is furthermore realistic at frequencies in the upper millimeter-wave range [3].

The eigenmode analysis starts with the well-known